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The SVIX index: a predictor of the market risk premium

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Resumo

A presente tese apresenta o cálculo e comparação de dois índices: o VIX (índice de volatilidade calculado pela Chicago Board Options Exchange) e o SVIX (apresentado no artigo científico de Martin (2017)[16]) para o período de janeiro de 2015 a abril de 2019. Ambos são índices de volatilidade cujos cálculos são baseados em opções sobre o índice S&P 500 e o seu estudo é referido como tendo importância para o estudo do retorno do mercado.

O índice VIX é calculado diariamente pela Chicago Board Options Exchange para representar a volatilidade implícita com base em opções. As opções utilizadas são europeias, out-of-the-money, sobre o índice S&P 500 (SPX). Para o cálculo deste índice, a Chicago Board Options Exchange estima a volatilidade implícita das referidas opções cuja maturidade é de, em média, 30 dias. O índice não se transaciona diretamente no mercado mas, através de produtos financeiros como futuros, opções ou Exchange Traded Funds (ETFs) é possível efetuar transações.

Após os cálculos efetuados em Matlab, para um horizonte de 30 dias, obtêm-se, neste estudo, resultados semelhantes aos publicados pela Chicago Board Options Exchange, especialmente se a comparação for realizada com o foco na direção das variações. Os resultados foram representados graficamente através de um gráfico que compara o VIX calculado em Matlab, para o referido horizonte de 30 dias, com os dados históricos fornecidos pela Chicago Board Options Exchange. Para completar a análise, foi também gerado um gráfico de diferenças entre os resultados das duas fontes (cálculo realizado para este estudo comparado com os dados disponíveis no mercado) que se espera que tenha um valor o mais próximo de zero que seja possível. Apesar de existirem diferenças não nulas para alguns dias, os resultados obtidos não são muito diferentes dos dados históricos. De acordo com os cálculos efetuados, as diferenças encontradas têm origem no facto de ter sido utilizada uma curva de taxas de juro de Nelson Siegel em vez da curva originalmente proposta pela Chicago Board Options Exchange, baseada em taxas "Constant Maturity Treasury" (CMT). Através de técnicas de interpolação e extrapolação é possível fazer a transformação necessária, de taxas calculadas para horizontes predefinidos em taxas de horizonte temporal "near" e "next", conforme requerido. Outra das diferenças foi a escolha da própria janela temporal em análise. Esta, foi também ligeiramente diferente da original apresentada por Martin (2017) [16] visto que, para os anos anteriores a 2015, nos dados fornecidos pela Chicago Board Options Exchange, existem mais tipos de opções além de "standard" e "weekly" (a utilização destes dois tipos de opções é descrita e replicada conforme a publicação CBOE (2018) [10]), não diretamente convertíveis nos dois tipos já mencionados, atualmente usados pela Chicago Board Options

Exchange no cálculo do índice VIX. Isto tornaria difícil obter informação comparável.

O índice VIX é uma ferramenta importante e deve ser estudado e compreendido para permitir um maior entendimento sobre a génese do índice SVIX, sendo este último o que mais contribui para as conclusões do estudo realizado nesta tese. No entanto, o próprio Martin (2017) [16] descreve e refere-se bastante ao índice VIX pela sua utilidade e pelas semelhanças e diferenças encontradas quando comparado com o índice SVIX. Esse padrão é também replicado aqui, sendo que o índice VIX serve de ponto de partida para a compreensão do SVIX. Através das equações apresentadas nos capítulos 2 e 3 desta tese, é possível compreender quais as várias etapas e componentes que constituem o cálculo destes índices, numa tentativa de os apresentar de uma forma comparável entre si, mas também não muito diferente da utilizada pelos autores, quer a CBOE para o VIX como Ian Martin para o SVIX.

Sobre o índice SVIX não existe tanta informação disponível como para o índice VIX, já que o primeiro índice mencionado se trata de uma abordagem descrita apenas teoricamente em 2017 por Ian Martin no seu artigo científico cujo título original é "what is the expected return on the market". O próprio título ilustra bastante bem a essência do artigo, que procura obter conclusões sobre a análise do prémio de risco de mercado. O índice SVIX ainda não tem observações históricas disponíveis no mercado mas poderá vir a ser implementado no futuro. Um dos grandes objetivos do artigo de Martin (2017) [16] é o de mostrar a relação existente entre o prémio de risco de mercado e o índice SVIX, que é apontada como uma das grandes vantagens da utilização e cálculo deste índice, relação essa também explorada nesta tese. O ponto de partida é uma identidade que associa o retorno esperado do mercado com a sua variância neutra ao risco. A condição de correlação negativa apresentada por Martin (2017) [16] também é de elevada importância para alcançar o objetivo do autor. Mais adiante no referido artigo de Ian Martin, é construída uma regressão que apresenta o índice SVIX como limite inferior do prémio de risco de mercado.

Na presente tese, os cálculos para obter os valores do índice SVIX foram efetuados para diferentes horizontes temporais e representados através de gráficos comparativos com o VIX observado pela Chicago Board Options Exchange. Martin (2017)[16] relaciona o limite inferior do prémio de risco de mercado para diferentes horizontes (30, 60, 90, 180 e 360 dias) com o índice SVIX, através de uma regressão para a qual são estimados dois parâmetros, para cada horizonte. Fazendo a distinção do propósito dos índices, o SVIX mede a volatilidade neutra ao risco do retorno do mercado enquanto que, o VIX, ao dar maior relevo às opções de venda out-of-the-money e menos às opções de compra out-of-the-money, mede a entropia neutra ao risco.

Além dos cálculos efetuados, para obter conclusões mais concretas e que podem ajudar a resumir a essência desta tese, foram analisados alguns indicadores econométricos.

Apresentam-se, de seguida, as conclusões do estudo realizado, lembrando o objetivo da tese de perceber se o índice SVIX é (ou não) um elemento importante para prever o prémio de risco de mercado. Antes disso, utilizando as comparações realizadas entre o índice VIX e o índice SVIX, conclui-se que, em média, para um horizonte de 30 dias, o valor do VIX é 0.16 mais elevado do que o SVIX, conforme previsto originalmente após a análise inicial do artigo de Martin (2017) [16]. Também é possível concluir que se podem calcular, a nível prático, ambos os índices, VIX e SVIX, com base nos preços de opções que podem ser obtidos no mercado e alcançar resultados semelhantes aos da CBOE e de Martin (2017)[16]. É de realçar a importância de estudar ambos, visto que, pelas suas diferenças, ambos contribuem para o estudo das características do mercado. Na análise aqui realizada percebe-se que, de facto, o índice SVIX pode ser considerado uma previsão para o prémio de risco de mercado. As conclusões sobre este facto foram um dos grandes objetivos deste estudo e estão devidamente suportadas pelo coeficiente de correlação de Pearson (que mede a correlação linear entre variáveis) de aproximadamente 0.422 (valor considerado moderado) e também pelo teste de causalidade de Granger que permite concluir que não se rejeita a hipótese nula de que a "lower bound" baseada no índice SVIX "causa-Granger" o prémio de risco de mercado. Este último indicador referido aponta para a mesma conclusão que também Martin (2017) [16] refere, visto que o teste de causalidade de Granger mostra que, neste caso, os valores passados do índice SVIX podem ser úteis para prever o prémio de risco de mercado.

Palavras-chave: Índices, Opções, Risco, SVIX, VIX

Abstract

In this thesis, both the Chicago Board Options Exchange volatility index (VIX) and the volatility index introduced by Martin (2017)[16] (SVIX) are presented, computed and compared for the period from January 2015 to April 2019. Martin (2017)[16] establishes a connection between the lower bound on the equity risk premium for different horizons (30, 60, 90, 180 and 360 days) and the SVIX index by building a regression for which two parameters are estimated, for each horizon. The SVIX index measures the risk neutral volatility of the market return while VIX gives more importance to the out-of-the-money put options and less to the out-of-the-money call options, as it measures the risk neutral entropy. In conclusion, it is possible to compute both, based on the option prices available in the market and obtain values that are similar to the ones obtained by CBOE and Martin (2017)[16]. It is important to see that both are relevant and worth studying because they both provide a contribution to the study of the market return variability. With this analysis it is evident that SVIX can indeed be considered a predictor of the market risk premium.

Keywords: Indexes, Options, Risk, SVIX, VIX

Contents

List of Figures

List of Tables

1. Introduction

The purpose of this thesis is to replicate the CBOE VIX index (Chicago Board Options Exchange's volatility index) and to replicate the SVIX index proposed by Martin (2017) [16]. The SVIX is proposed as a lower bound for the equity market risk premium.

The VIX index is the implied volatility index published by the Chicago Board Options Exchange (CBOE). It is computed based on an European-style portfolio of out-of-the-money (OTM) options on the S&P 500 index (SPX) by estimating the implied volatility of these options at an average expiration of 30 days and it is negatively correlated with SPX. The VIX index is not bought or sold directly but it is traded through derivative products: futures, options or Exchange-Traded Funds (ETFs).

In Chapter 2, the CBOE VIX index is analysed, replicated and compared to the observed VIX values.

In Chapter 3, all the relevant formulas needed to understand what the SVIX index is are stated and the projections done in Matlab are presented as a preparation for the conclusions that include the comparison with the VIX index. The calculations done for the SVIX index are done for the same setup that Martin (2017)[16] considers.

The necessary option data was directly requested from CBOE through ISCTE's Business Research Unit. The scientific papers that are mentioned throughout this work were chosen to be read because they have information that can be helpful to understand, and the main two are: CBOE (2018)[10] and Martin (2017) [16]. CBOE (2018)[10] contains the steps to use the method that CBOE currently puts into practice in order to compute the VIX index. Martin (2017)[16] shows a different approach, by showing the steps to compute the SVIX index for different horizons and then by building a regression that relates the Lower Bound on the Equity Premium to the Equity Risk Premium itself. These two approaches are tested in this thesis for the period between January 2nd, 2015 and April 30th, 2019 as the option data for the previous years is not clear to work with because the option types considered by CBOE for the previous years are more and are different from the current ones, not including only "standard" and "weekly" options. These option types were eventually merged into standard and weekly but there is no direct match that allows us to use this data in the same way as the one from 2015 to 2019. The problem of using the previous years' data was that the search of the near- and next-terms options data was impacted. The formulas used are applied exactly as it is explained in the selected scientific papers. Regarding the yield curve, a Nelson Siegel yield curve is used to determine the risk free rates to be used. Both interpolation and extrapolation techniques are applied to move from rates of prearranged terms (1 week, 1 month, 2 months, 3 months, 6 months and 1 year) to the near- and next-term rates as needed.

The main tool used is Matlab, following the steps outlined below:

1 - Based on the CBOE VIX White Paper CBOE (2018)[10], create a script that computes daily VIX index values for the dates provided;

2 - Run the script using the data provided by CBOE, from 2015 to 2019. Compare the data with CBOE's VIX observations;

3 - Modify the initial VIX script in order to compute the SVIX index for different horizons.

There is a lot of information about VIX as this volatility index is currently used. For example, CBOE has published two documents — CBOE (2018)[10] and CBOE(2019)[11] — with the steps for the calculation. Furthermore, Macrotrends (2019)[3] publishes daily VIX observations. An index currently called VXO is also known as "the old VIX index" and the calculation is described by Carr and Wu (2006)[9]. It uses Black-Scholes implied volatilities of S&P 100 index options (OEX), instead of market prices. In the same paper, the authors describe the current procedure to compute VIX and argue that VIX can be seen as "the variance swap rate quoted in volatility percentage points" and that it is "the price of a linear portfolio of options" (Carr and Wu (2006, pages 15-16) [9]).

There is not much information available about the SVIX index as it has not been adopted so far by the market and there are no recorded observations or historical data given by the market. SVIX is an innovating index created by Martin (2017)[16], starting with an identity that links the expect return on the market with its risk-neutral variance. The latter can be measured from index option prices. The mentioned identity, together with the negative correlation condition, allows Martin (2017) [16] to derive a lower bound on the equity premium in terms of the SVIX index. The index can be interpreted as the equity premium anticipated by an unconstrained rational investor with log utility that is fully investing in the market. Martin (2017, page 369)[16] refers that the equity premium is slightly different from what the literature has acknowledged:"it is more volatile, more right-skewed and fluctuates at a higher frequency".

Li (2019)[15] applied some of the procedures that are shown in subsection 2.3. and that are very helpful to understand if the calculations done are in accordance with CBOE's data.

2. VIX

2.1. VIX as an implied volatility index

CBOE (2018)[10] introduces the VIX index starting with its background. It is interesting to notice that it was initially created to "measure the market's expectation of 30-day volatility implied by at-the-money S&P 100 Index option prices", as mentioned in CBOE (2018, page 3)[10]. After this, the index became the premier benchmark for the volatility of the U.S. stock market. Ten years later, CBOE and Goldman Sachs adapted the VIX index to a new way of measuring expected volatility. The VIX index measures expected volatility based on aggregating the weighted prices of a large range of European-style puts and calls with different strikes and a time-to-maturity around 30 days, all on the S&P 500 Index (SPX). Another update, done in 2014, allowed CBOE to include SPX "weekly" options as well, which contributes to a higher accuracy in matching the 30-day target as it is possible to use options with more than 23 days and less than 37 days to expiration. The "standard" SPX options expire in the 3rd Friday of the month and would not be as good for the interpolation.

VIX index values are important for risk management, financial theory and volatility trading but the index is not traded itself. It is traded through other financial assets such as derivative products (futures, options and ETFs, for example). The first VIX futures contract to be traded in the market was introduced in 2004, two years before the launch of VIX options (that became very popular). The negative correlation that relates volatility with stock market returns shows evidence of the benefit of including volatility in an investment portfolio.

According to Martin (2017)[16], as the options used to compute VIX are weighted by the inverse of the squared strike, VIX highlights out-of-the-money puts and places less weight on out-of-the-money calls. It means that it also places more weight on left-tail events.

VIX can be expressed as the squared root of:

$$\sigma^2(t,T) = \frac{2}{T-t} \sum_{i=-n}^{N} \frac{\Delta K_i}{K_i^2} \frac{1}{P(t,T)} \overline{O}_t(S_t, K_i, T) - \frac{1}{T-t}\left[\frac{F(t,T) - K_0}{K_0}\right]^2, \tag{1}$$

where $T - t = \frac{30}{365}$, $F(t,T) = S_t \; e^{(r-q)(T-t)}$, S_t is the SPX level at time-t, K_0 is the first strike below $F(t,T)$, $P(t,T)$ is the time-t discount factor $e^{-r(T-t)}$, r representing the risk-free discount rate and q the dividend yield, $\overline{O}_t(S_t, K_i, T)$ is the time-t price of an OTM Forward option on the SPX index with strike K_i and maturity at time T (except for K_0) , i.e.

$$\overline{O}_t(S_t, K_i, T) := \begin{cases} p_t(S_t, K_i, T) \times \mathbb{1}_{\{F(t,T)>K_i\}} + c_t(S_t, K_i, T) \times \mathbb{1}_{\{F(t,T)<K_i\}} <= i \neq 0 \\ \frac{c_t(S_t, K_0, T) \; + \; p_t(S_t, K_0, T)}{2} \qquad <= i = 0 \end{cases}, \tag{1a}$$

$c_t(S_t, K_i, T)$ and $p_t(S_t, K_i, T)$ are, respectively, the time-t prices of European-style calls and puts on the asset S with strike K_i and maturity at time T,

$$\Delta K_i = \begin{cases} K_N - K_{N-1} \quad <= i = N \\ \frac{K_{i+1} - K_{i-1}}{2} \qquad <= -n < i < N \\ K_{-n+1} - K_{-n} <= i = -n \end{cases}, \tag{1b}$$

K_{-n} is the lowest strike price available with a non-zero bid and K_N is the highest strike price with a non-zero bid.

Appendix A.2 shows that the continuous-time limit of equation (1) is:

$$\sigma^2(t,T) \approx \frac{2}{T-t} e^{r(T-t)} \int_0^\infty \frac{1}{K^2} O_t(S_t, K, T) \; dK, \tag{2}$$

where

$$O_t(S_t, K_i, T) = p_t(S_t, K_i, T) \times \mathbb{1}_{\{F(t,T)>K_i\}} + c_t(S_t, K_i, T) \times \mathbb{1}_{\{F(t,T)<K_i\}}. \tag{2a}$$

2.2. Methodology

To compute the VIX index, as it measures the 30-day expected volatility, near and next-term options are used. The near-term options have their maturity date on the first Friday more than 23 days after the current day while the next-term options have their maturity one week after the near-term. There are some rules and steps to compute VIX index values:

1. The selected options are out-of-the-money SPX calls and puts centred around an at-the-money strike price K_0. Only SPX options with non-zero bid prices are used for the calculation;

2. Determine $F(t,T)$ (the forward SPX level derived from index option prices);

3. Determine K_0 (the strike price immediately below $F(t,T)$);

4. Select out-of-the-money put options with strike prices below K_0 and move to successively lower strike prices;

5. Exclude options that have a bid price equal to zero and after two in a row (each representing a strike price), do not consider any lower strikes;

6. Select out-of-the-money call options with strike prices above K_0 and move to successively higher strike prices;

7. Exclude options that have a bid price equal to zero and after two in a row, do not consider any higher strikes;

8. Compute the mid-prices (i.e. average bid-ask) for each option selected;

9. For K_0, use the average of the put and call options prices;

10. Compute the contribution of each option to the VIX value using $\frac{\Delta K}{K^2}\frac{1}{P(t,T)}O_t(S_t,K,T)$ (where K is the strike price, $P(t,T)$ is the discount factor and $O_t(S_t,K,T)$ is the mid-quote price for the given strike);

11. Sum the contributions and multiply by $\frac{2}{T-t}$ (where $T-t$ is the time to expiration);

12. Compute $\frac{1}{T-t}\times\left(\frac{F(t,T)}{K_0}-1\right)^2$;

13. Compute the squared root of the 30-day weighted average of the volatility and multiply it by 100.

As the "standard" SPX options expire on the 3rd Friday of each month, these are used when the expiration date fits into the above-mentioned window. When this is not possible, "weekly" SPX options that expire every Friday, except the 3rd Friday of each month, are used for the calculation. Hence, the "standard" options can only be used for one of the near and next terms.

Note also that if there is a holiday on a specific Friday and, thus, there is no data available for that date, the options expire either on the closest Thursday or Saturday.

In order to calculate VIX, out-of-the-money SPX calls and out-of-the-money SPX puts centered around an at-the-money strike price, K_0, are used. Only SPX options with non-zero bid prices are used in the VIX calculation. The range of options that can be used may expand and contract as volatility rises and falls, so the number of options used to compute the VIX index may also vary.

The process starts by computing the forward SPX level, $F(t,T)$, through put-call parity, after identifying the smallest difference between the call and put prices, based on a bid-ask average. Then, K_0 is determined and all put options with strike prices smaller than K_0, excluding any with a zero bid price, are selected until there are two consecutive strikes with zero bid prices. Out-of-the-money call options with strike prices bigger than K_0 are used based on the same procedure.

The VIX index is given by:

$$VIX(t,T) = 100 \times \sqrt{\left[T_1\sigma^2(t,T_1)\left(\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right) + T_2\sigma^2(t,T_2)\left(\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right)\right] \times \frac{N_{365}}{N_{30}}}, \quad (3)$$

where $\sigma(t,T_1)$ is the near-team volatility, $\sigma(t,T_2)$ is the next-term volatility, $T_1 < T < T_2$, N_{T_1} represents the number of minutes to the settlement of the near-term options while N_{T_2} represents the same for the next-term options, N_{30} is the number of minutes in 30 days, N_{365} is the number of minutes in 365 days (one year).

2.3. Empirical analysis

With all the relevant formulas already detailed in the previous sections, the graph produced by the Matlab program is compared in Figure 1 to the one built by CBOE. It is possible to see the likeness between both lines in Figure 1. Moreover, Table 1 shows the results of applying the same procedures of Panel A and C from Li (2019, Table I)[15] to assess the consistency between the computed estimate and the CBOE VIX index. The results are very positive as the computed differences are all very small (as per the last line of Table 1).

Table 1.: Quantiles estimates of observed daily CBOE's VIX index closing values (CBOE (2020)[1] and the calculated daily VIX from January 2015 to April 2019

VIX	Minimum	25%	50%	75%	Maximum
Historical	9.14	12.09	13.84	16.94	40.74
Calculated	9.17	12.05	13.78	16.92	41.35
Difference	-0.03	0.04	0.06	0.02	-0.61

The two-sample Kolmogrov-Smirnov test was done for both distributions of the VIX daily observations, the one projected in Matlab and the one published by CBOE. It resulted in a p-value of 0.99999866 and the hypothesis test result was zero, meaning that the null hypothesis, that both samples are from the same continuous distribution, is not rejected (at the default 5% significance level that the kstest2 function considers).

The results were obtained based on the inputs used, subject to some restrictions that are explained here. Only options with maturities after 2015 were included as there were many limitations with the data provided by CBOE for the previous years, including: there were many different types of options, not only "standard" or "weekly" and it is not clear which were used by CBOE, not allowing us to select the correct ones with a high level of confidence; some of the data of the previous years seems corrupted as for some dates it was not even possible to apply the methodology and obtain a VIX value. The available data from 2004 to 2014 is monthly-based instead of weekly, and this prevents us from organizing the near- and next-term sets. There are more than 25 years of historical prices but based on different assumptions, which makes it very hard to have a well defined approach to use the data.

As usual, risk-free rates used for the computation of VIX values are yields based on the US Treasury yield curve rates ("Constant Maturity Treasury" or CMTs), but smoothed through the Nelson-Siegel approach (Nelson and Siegel (1987)[18]).

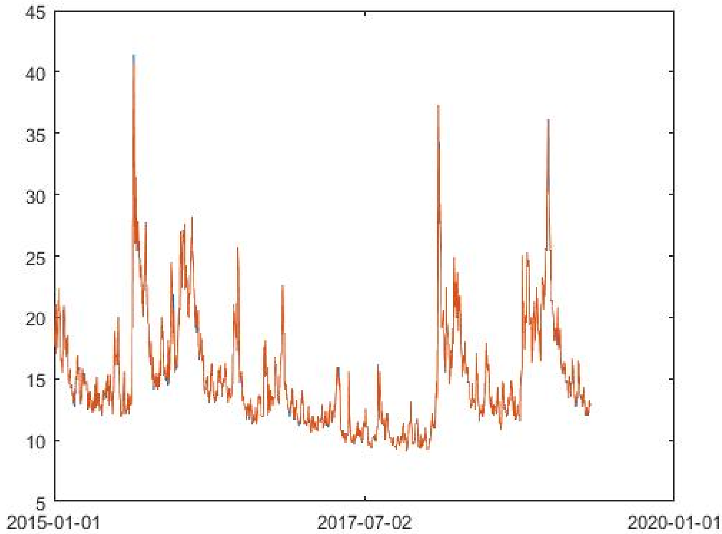

Figure 1.: VIX observed (in red) in the market compared to the one computed in Matlab (in blue)

In Figure 1, it is not easy to identify differences between the two lines. For this reason, further analysis were perfomed like the above mentioned two-sample Kolmogrov-Smirnov test and the quantiles comparisons. By checking each day and computing the differences, it is possible to conclude that, on average, Macrotrends observations are 0.033861% higher than the results obtained in Matlab. This tiny difference can be justified by the previously mentioned interest rate temporal series that was used in the calculations, which is different from the one used by CBOE (2018)[10].

3. SVIX

3.1. SVIX as a lower bound for the Equity Risk Premium

Martin (2017)[16] derives a tight lower bound for the equity risk premium in terms of a volatility index named SVIX and computed from index option prices. The bound implies that the equity premium is extremely volatile. The time-series average of the lower bound is about 5%. Predictive regressions are run to confirm that the hypothesis — that the SVIX index is a proxy for the equity risk premium — is not rejected by the data.

Martin (2017)[16] starts with an identity that relates the market's expected return to its risk-neutral variance. Assuming no arbitrage, risk-neutral variance is measured unambiguously from index option prices using the new SVIX volatility index. The identity is used to derive a lower bound on the equity premium in terms of the SVIX index. The main contribution of Martin (2017)[16] is that he argues that the SVIX index provides a direct measure of the equity premium. SVIX is motivated by asset pricing theory, it does not require parameter estimation and it is an asset price so it avoids using not up to date accounting data. These facts allow the equity premium to be measured in real time. However, the expected returns measured through the SVIX index are short-run returns while the ones provided by valuation ratios are long-run returns.

Hereafter, R_T is the asset return, assumed by Martin (2017)[16] to be the return on the S&P 500 index. Risk-neutral variance is obtained based on given time-t prices of S&P 500 index options and it is a directly observable quantity. It is possible to relate the unobservable equity premium to the risk-neutral variance.

3.2. Methodology

Martin (2017, page 370)[16] states that the SVIX index "aims to measure short-run expected returns". This is one of the main diferences when compared to other valuation ratios. It is also important to understand that he explains all the steps and equations with the goal of showing the lnk between the SVIX index and the Equity Risk Permium. Both the choice and use of the data, together with the connections between variables are well explained by Martin (2017)[16]. As explained by Martin (2017, Appendix A)[16], the data used to compute the SVIX index is compiled in a similar way as the one used for the VIX. The main steps are:

1. Gather the closing price of S&P 500 index options, the expiration date, the strike price, the highest closing bid and the lowest closing ask of all options with less than 550 days to expiry (call and put options) — this allows us to use the same data that was used for the VIX;

2. Delete all duplicated lines;

3. Choose the call or put options with the lowest mid price for each strike;

4. Remove the options that have the highest closing bid of zero;

5. Delete all Quarterly options — not considered in this case as between 2015 and 2019 there are only standard and weekly options.

After the above steps are put into practice, the filtered data can be used to compute the lower bound on a specific horizon. This horizon will lean on the expiration dates of the options traded on that day. Then, the same logic used by CBOE for VIX, of the "near" and "next" terms with no more than 7 days apart from the target date, is reproduced. Martin (2017)[16] does it, through linear interpolation, for 30, 60, 90, 180 and 360 days. It is important to understand the connection between the variables used and how they all have their role in the final calculations. The next paragraphs allow us to know more about the process used by Martin (2017) and the way the variables are linked [16].

In what follows, let $R_{f,t,T}$ be the risk-free return over the time-period [t,T] or $R_{f,t,T} = \frac{1}{P(t,T)}$, while M represents the stochastic discount factor (SDF) that prices the time-T payoff at time t. Hence,

$$1 = {R_{f,t,T}}^{-1} \times \mathbb{E}_{\mathbb{Q}}(R_T/\mathbb{F}_t) = \mathbb{E}_{\mathbb{P}}(M_T R_T/\mathbb{F}_t) \tag{4}$$

where $\mathbb{P}$ and $\mathbb{Q}$ denote, respectively, the physical and the risk-neutral probability measures, while F_t is the time-t σ-algebra for the economy under analysis. Martin (2017)[16] mentions that the equity premium is relevant to assess the risk premium (for arbitrary assets in the context of the capital asset pricing model) and, at the same time, the time variation in the equity premium is very often studied when the subject is excess volatility. Based on the above equation, Martin (2017, equation 4)[16] is able to link expected returns and risk-neutral variance:

$$
\begin{aligned}
\mathbb{E}_{\mathbb{P}}(R_T/\mathbb{F}_t) - R_{f,t,T} &= \left[\mathbb{E}_{\mathbb{P}}(M_T {R_T}^2/\mathbb{F}_t) - R_{f,t,T}\right] - \left[\mathbb{E}_{\mathbb{P}}(M_T {R_T}^2/\mathbb{F}_t) - \mathbb{E}_{\mathbb{P}}(R_T/\mathbb{F}_t)\right] \\
&= \left[{R_{f,t,T}}^{-1}\mathbb{E}_{\mathbb{Q}}({R_T}^2/\mathbb{F}_t) - \mathbb{E}_{\mathbb{Q}}(R_T/\mathbb{F}_t)\right] \\
&- \left[\mathbb{E}_{\mathbb{P}}(M_T {R_T}^2/\mathbb{F}_t) - \mathbb{E}_{\mathbb{P}}(M_T R_T/\mathbb{F}_t) \times \mathbb{E}_{\mathbb{P}}(R_T/\mathbb{F}_t)\right] \\
&= {R_{f,t,T}}^{-1} \times var_{\mathbb{Q}}(R_T/\mathbb{F}_t) - cov_{\mathbb{P}}(M_T R_T, R_T/\mathbb{F}_t)
\end{aligned} \tag{5}
$$

Martin (2017)[16] argues that:

$$cov_{\mathbb{P}}(M_T R_T, R_T) \leq 0 \tag{6}$$

for most asset pricing models and labels the previous inequatily as a Negative Correlation Condition (NCC). Combining the two previous equations, Martin (2017, equation 5)[16] arrives at his equity risk premium lower bound:

$$\mathbb{E}_{\mathbb{P}}(R_T/\mathbb{F}_t) - R_{f,t,T} \geq \frac{1}{R_{f,t,T}} \times var_{\mathbb{Q}}(R_T/\mathbb{F}_t) \tag{7}$$

At this stage, Martin (2017)[16] explains that the approach followed by Merton (1980) [17] is useful for a better understanding of his own approach. Merton (1980) [17] assumes that the level of the stock index follows a geometric Brownian motion and Martin (2017)[16] shows that he does not need to consider this assumption. Moving forward with this geometric Brownian motion, the generalization allows to relate the risk premium to the risk-neutral varance and this is a great advantage because it is possible to directly compute forward-looking risk-neutral variance at time-t based on asset prices at time t as shown by Martin (2017)[16]. The approach followed by Martin (2017) [16] also allows to expand Merton (1980)[17] calculation of the instantaneous risk premium to a more general reality, not only considering the assumption of Black and Scholes (1973)[6] that prices follow geometric Brownian motions.

In order to measure risk-neutral variance, Martin (2017)[16] starts by assuming that the prices of European-style call and put options with maturity at time-T on the asset with return R_T are observable at all strikes K. Moreover, call and put prices are convex functions of strike (to avoid any arbitrage opportunities) and, that the forward price $F(t, T)$ can be computed through the observation of the strike at which call and put prices are equal, meaning that $F(t, T)$ is the unique solution x of $c_t(x, K, T) = p_t(x, K, T)$ and, the forward price can be "backed out" from time-t option prices representing by S_T the time-T price of the underlying asset, then

$$F(t,T) = \mathbb{E}_{\mathbb{Q}}(S_T/\mathbb{F}_t). \tag{8}$$

Assuming that the dividends generated between times t and T are known at time-t and paid at time-T, and considering S_t as the time-t price of the market index, then $R_T = \frac{S_T}{S_t}$, and it is possible to expand the right-hand side of equation (7):

$$\begin{aligned} \frac{1}{R_{f,t,T}} var_{\mathbb{Q}}(R_T/\mathbb{F}_t) &= \frac{1}{{S_t}^2}\left[\frac{1}{R_{f,t,T}}\mathbb{E}_{\mathbb{Q}}({S_T}^2/\mathbb{F}_t) - \frac{1}{R_{f,t,T}}\Big(\mathbb{E}_{\mathbb{Q}}(S_T/\mathbb{F}_t)\Big)^2\right] \\ &= \frac{1}{{S_t}^2}\left[\frac{1}{R_{f,t,T}}\mathbb{E}_{\mathbb{Q}}({S_T}^2/\mathbb{F}_t) - \frac{1}{R_{f,t,T}}\Big(F(t,T)\Big)^2\right], \end{aligned} \tag{9}$$

where the last line follows from equation (8). On the other hand, $\frac{1}{R_{f,t,T}}\mathbb{E}_{\mathbb{Q}}({S_T}^2/\mathbb{F}_t)$ is the price of a claim to ${S_T}^2$ paid at time-T, i.e., the price of the "squared contract". Martin (2017, Figure III)[16] illustrates this by showing both the idealized payoff ${S_T}^2$ (represented by a dotted line) and the payoff on a portfolio with two options of each strike (represented by a solid line). The lines have an almost equal behavior, with the payoff on the portfolio being exactly ${S_T}^2$ at the integer values of the payoff ${S_T}^2$. Equation (10) represents this by considering, on its left-hand side, the payoff ${S_T}^2$ and, on its right-hand side, the payoff on the portfolio:

$$\frac{1}{R_{f,t,T}}\mathbb{E}_{\mathbb{Q}}({S_T}^2/\mathbb{F}_t) \approx 2\sum_K c_t(S_t,K,T). \tag{10}$$

To obtain an exact expression, as for any $x \geq 0$, $x^2 = 2\int_0^\infty max\{0, x-K\}dK$, with $x = S_T$, under risk-neutral expectations,

$$\frac{1}{R_{f,t,T}}\mathbb{E}_{\mathbb{Q}}({S_T}^2/\mathbb{F}_t) = 2\int_0^\infty \frac{1}{R_{f,t,T}}\mathbb{E}_{\mathbb{Q}}(max\{0, S_T - K\}/\mathbb{F}_t)dK = 2\int_0^\infty c_t(S_t,K,T)dK. \tag{11}$$

Combining the equations (9) and (11),

$$\frac{1}{R_{f,t,T}} var_Q(R_T/F_t) = \frac{1}{{S_t}^2}\left[2\int_0^\infty c_t(S_t,K,T)dK - \frac{F(t,T)^2}{R_{f,t,T}}\right]. \tag{12}$$

Due to the usual illiquidity of the deep-in-the-money call options, it can be useful to splt the range of integration into two, and, using the put-call parity formula, replace the in-the-money call prices with out-of-the-money put prices:

$$\begin{aligned}\int_0^\infty c_t(S_t,K,T)dK &= \int_0^{F(t,T)} \left[p_t(S_t,K,T) + \frac{1}{R_{f,t,T}}(F(t,T)-K)\right]dK \\ &+ \int_{F(t,T)}^\infty c_t(S_t,K,T)dK \\ &= \int_0^{F(t,T)} p_t(S_t,K,T)dK + \frac{F(t,T)^2}{2R_{f,t,T}} + \int_{F(t,T)}^\infty c_t(S_t,K,T)dK.\end{aligned} \tag{13}$$

Equation (13) combined with equation (12) yields:

$$\frac{1}{R_{f,t,T}} var_Q(R_T/F_t) = \frac{2}{{S_t}^2}\left[\int_0^{F(t,T)} p_t(S_t,K,T)dK + \int_{F(t,T)}^\infty c_t(S_t,K,T)dK\right]. \tag{14}$$

The right-hand side of equation (14) is very important to the definition of the VIX index. At this point, the SVIX index formula will be introduced below to emphasize that the SVIX index is calculated at time t based on the prices of options with maturity at time-T and it measures the annualized risk-neutral variance of the realized excess return from times t to T (Appendix A.2 shows how to compute the SVIX index in discrete time):

$$SVIX(t,T)^2 := \frac{2}{(T-t)R_{f,t,T}{S_t}^2}\left[\int_0^{F_{t,T}} p_t(S_t,K,T)dK + \int_{F(t,T)}^\infty c_t(S_t,K,T)dK\right]. \tag{15}$$

Comparing equations (14) and (15),

$$SVIX(t,T)^2 = \frac{1}{T-t} var_{\mathbb{Q}}\left(\frac{R_T}{R_{f,t,T}}/\mathbb{F}_t\right). \tag{16}$$

In subsection A.1 of the Appendix, the conditions for the Negative Correlation Condition (equation (6)) to hold are explained. Using equation (14) and inequality (7) it is possible to establish a lower bound on the expected excess return of an asset that complies with the Negative Correlation Condition (NCC):

$$\mathbb{E}_{\mathbb{P}}(R_T/\mathbb{F}_t) - R_{f,t,T} \geq \frac{2}{{S_t}^2}\left[\int_0^{F(t,T)} p_t(S_t,K,T)dK + \int_{F(t,T)}^\infty c_t(S_t,K,T)dK\right]. \tag{17}$$

Combining equations (7) and (15), the lower bound on the expected excess return of any asset that obeys the NCC, in terms of the SVIX index, is equal to:

$$\frac{1}{T-t}(\mathbb{E}_{\mathbb{P}}(R_T/\mathbb{F}_t) - R_{f,t,T}) \geq R_{f,t,T} \times SVIX(t,T)^2. \quad (18)$$

Martin (2017)[16] argues that the lower bound on the equity premium may be fairly tight as the mean of the lower bound over his sample is 5% at the monthly horizon. He also applies the bound to the S&P 500. Through Martin (2017, Figure IV)[16], he shows the lower bound for different horizons and then, in Martin (2017, Table I)[16], the mean, standard deviation and some quantiles of the distribution of the lower bound are shown for horizons from one month to one year. The bound is applied to the case of the S&P 500, based on a time seres of the lower bound built using OptionMetrics data. This exercise will be replicated and detailed in the subsection 3.3.

3.3. Empirical analysis

Martin (2017, Figure IV)[16], showing the SVIX index at different horizons (mid prices used to compute SVIX), is replicated below, in Figures 2, 3, 4, 5 and 6. One of the main differences between the two methodologies is that Martin uses different horizons to compute SVIX while CBOE focus on the 30-day horizon for the VIX calculation. In the following figures, the graphs that represent a comparison between 30-day historical VIX (red line) and different term SVIX calculations - 30-day, 60-day, 90-day, 180-day and 360-day (blue line) is made.

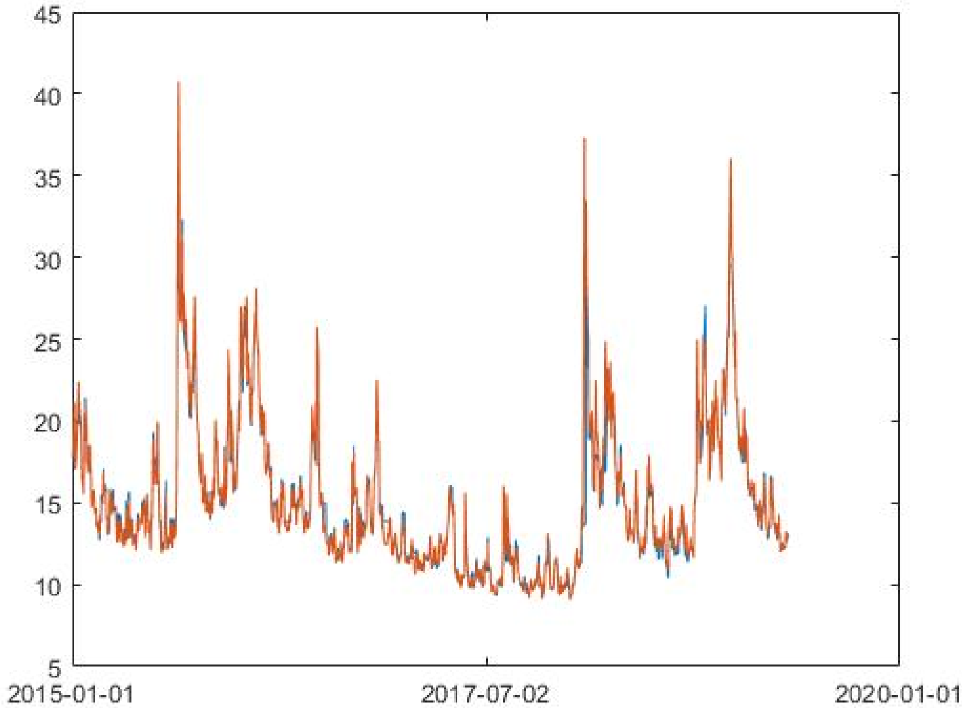

Figure 2.: VIX observed in the market (represented by the red line) compared to the 30-day horizon SVIX computed in Matlab (represented by the blue line)

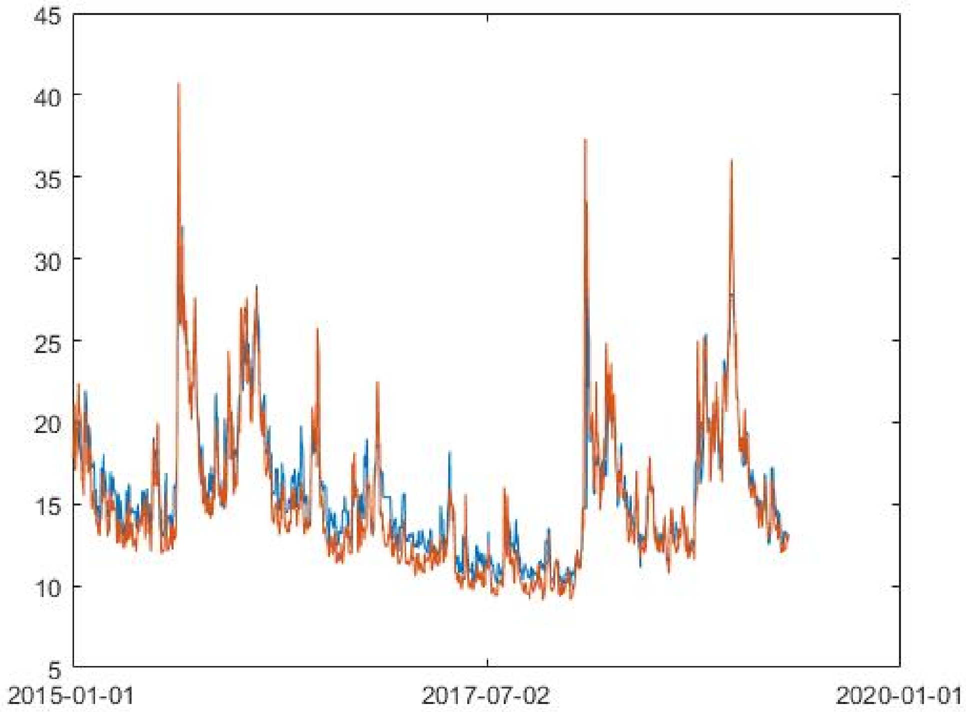

Figure 3.: VIX observed in the market (represented by the red line) compared to the 60-day horizon SVIX computed in Matlab (represented by the blue line)

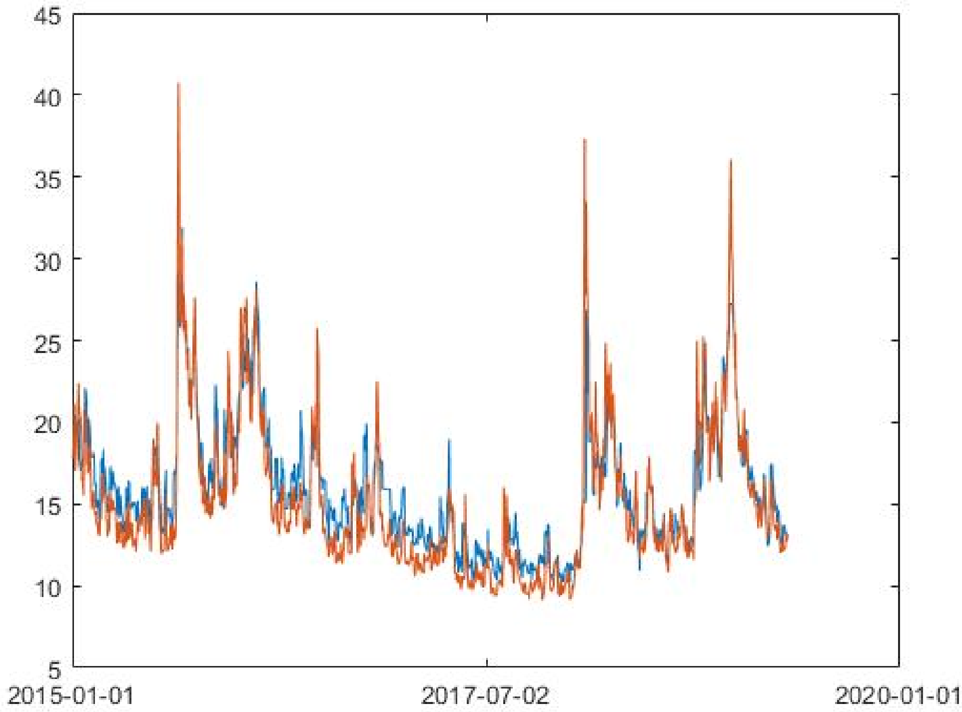

Figure 4.: VIX observed in the market (represented by the red line) compared to the 90-day horizon SVIX computed in Matlab (represented by the blue line)

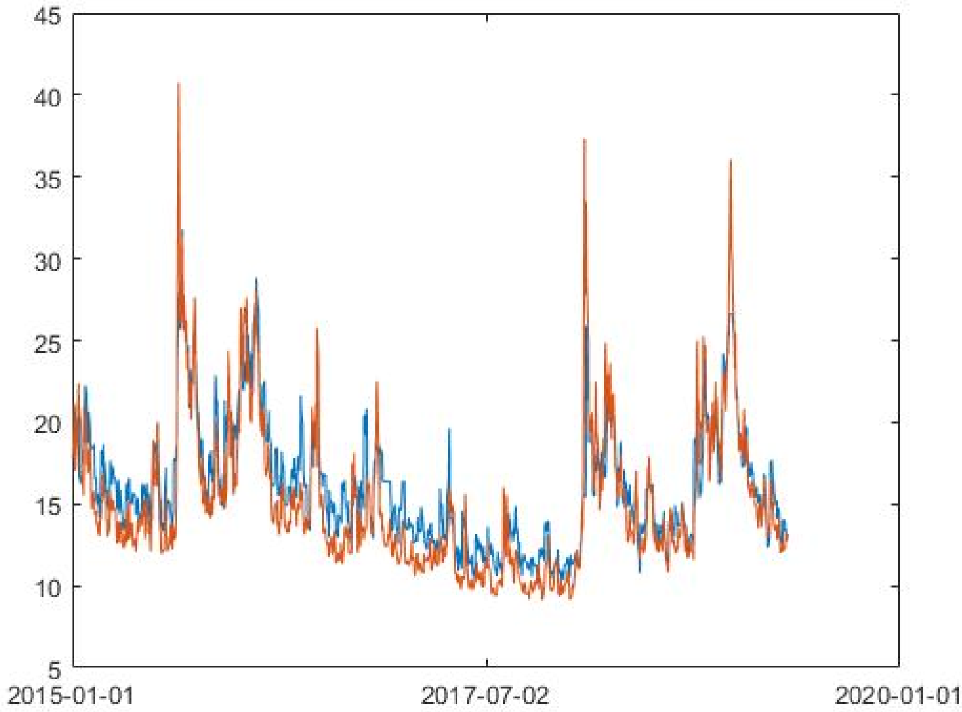

Figure 5.: VIX observed in the market (represented by the red line) compared to the 180-day horizon SVIX computed in Matlab (represented by the blue line)

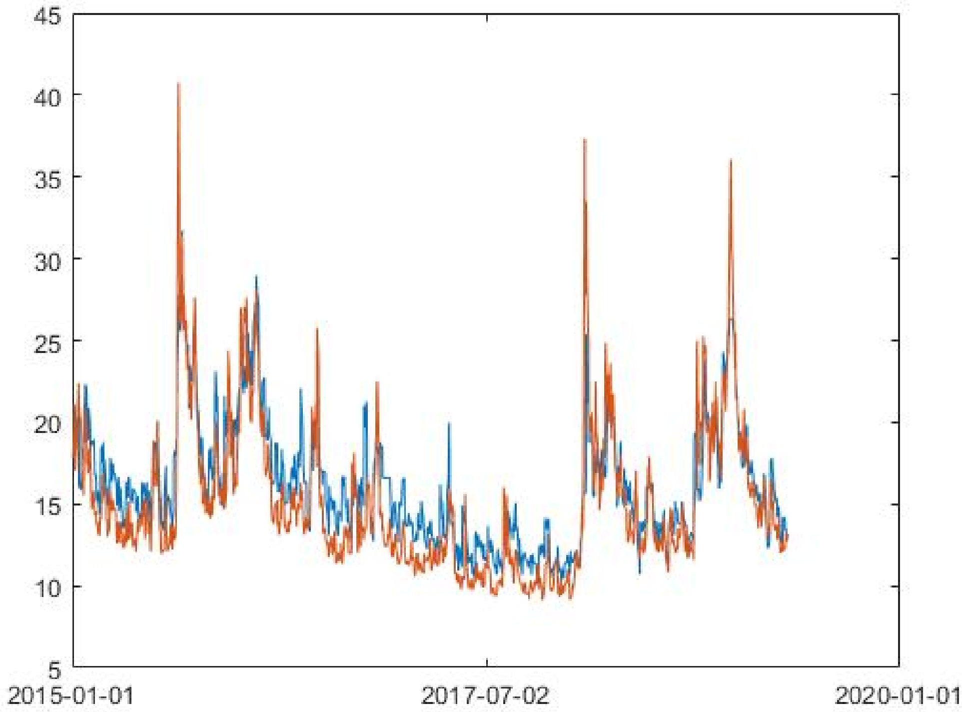

Figure 6.: VIX observed in the market (represented by the red line) compared to the 360-day horizon SVIX computed in Matlab (represented by the blue line)

Figures 2 to 6 show that for higher SVIX terms, there are higher differences between the 30-day historical VIX and the computed SVIX. The blue lines representing the SVIX for the higher terms seem to be more frequently above the red lines.

Martin (2017, Table I)[16] computes the mean, standard deviation, skewness, excess kurtosis and quantiles of the SVIX index, at various horizons (annualized and measured in %) and is replicated through a Matlab code as well. The results are shown in Table 2:

Table 2.: Mean, Standard Deviation, Skewness, Excess Kurtosis and Quantiles of the SVIX index at Various Horizons

Horizon	Mean	Std. Dev.	Skew	Kurt	Min	1%	10%	25%	50%	75%	90%	99%	Max
1 month	14,956	4,157	1,294	4,974	9,056	9,504	10,457	11,998	13,928	16,853	20,582	27,897	35,387
2 months	15,634	3,842	1,098	4,183	9,902	10,273	11,302	12,875	14,784	17,535	20,931	27,614	32,010
3 months	15,844	3,770	1,033	3,966	10,038	10,403	11,520	13,092	15,095	17,796	20,926	27,217	31,902
6 months	16,047	3,715	0,973	3,793	10,173	10,565	11,727	13,361	15,384	18,111	21,081	26,919	31,794
12 months	16,146	3,694	0,947	3,725	10,153	10,645	11,846	13,481	15,460	18,222	21,060	26,880	31,740

All of the measures of statistical dispersion mentioned in the Table 2 tend to fluctuate in the same direction that was observed by Martin (2017)[16], except for the average, that varies in the opposite direction.

In order to check Martin (2017)[16] theory of the difference between VIX and SVIX, Figure 7 represents the difference between VIX and SVIX for a 30-day horizon. The average is 0.16 which seems to point that VIX is higher than SVIX, as referred in Martin (2017, page 402)[16].

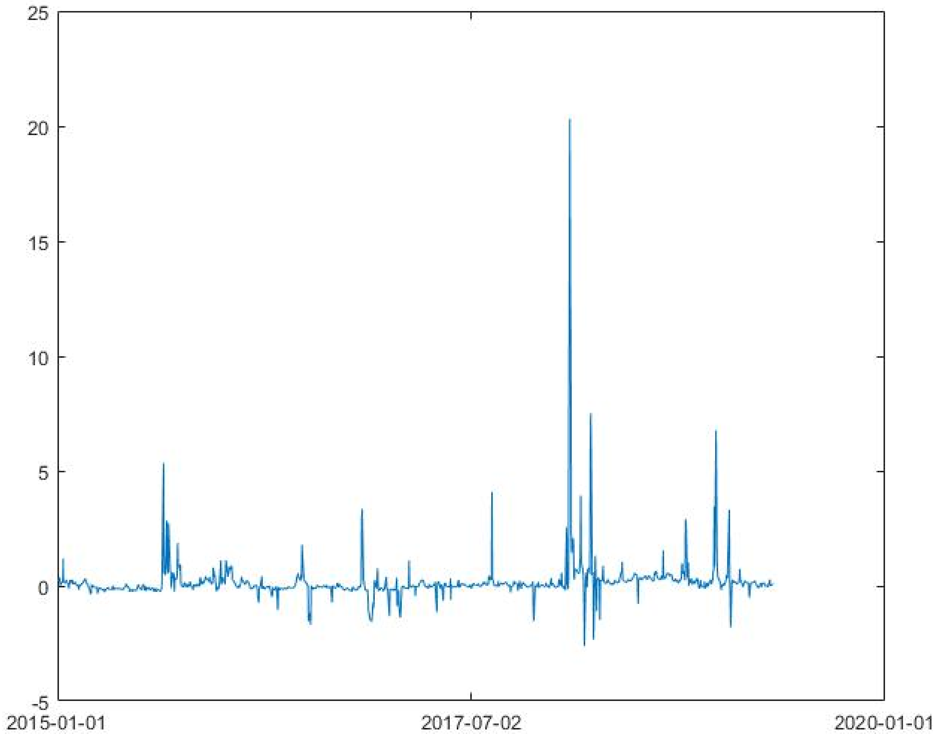

Figure 7.: Computed differences between 30-day historical VIX and 30-day computed SVIX

The Lower Bound based on the SVIX index for a 12-month horizon was compared, on a monthly basis, from January 2015 to April 2019, with the Equity Risk Premium. The goal was to check if the SVIX index can be considered a proxy for the Equity Risk Premium. The Equity Risk Premium data used was obtained from Damadoran (2020)[2]. In Figure 8, it is possible to see the behaviors of both the Equity Risk Premium and the Lower Bound and confirm that the Lower Bound line is always below the Equity Risk Premium line, except for two peaks.

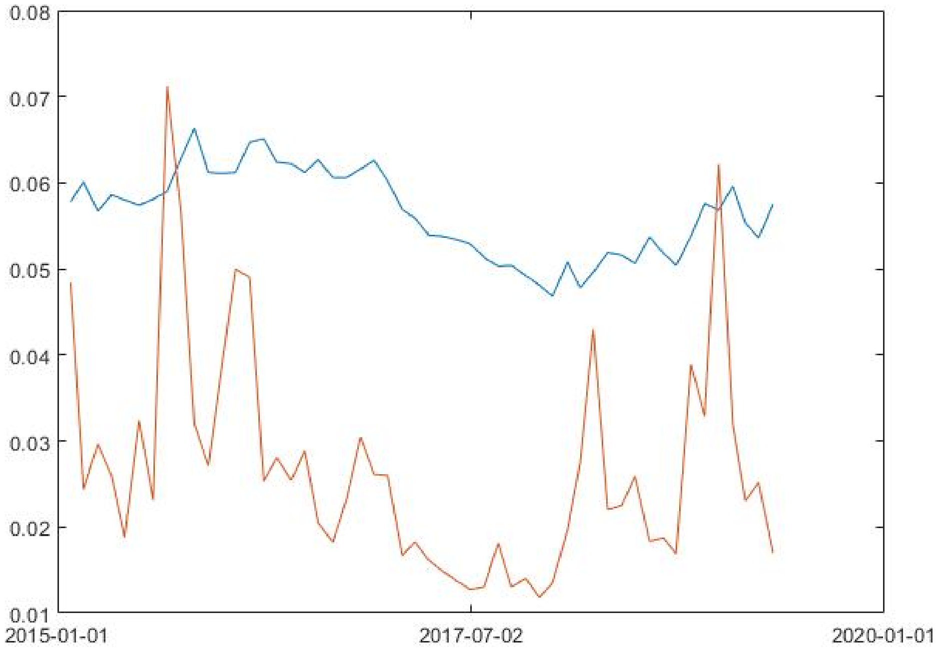

Figure 8.: Computed 12-month Lower Bound (in red) and the Equity Risk Premium (in blue) - Damadoran (2020)[2]

It is also important to analyse the variables based on econometric models to achieve a conclusion regarding the title of the thesis and undersand if the SVIX index is indeed a predictor of the market risk premium.

The Pearson correlation coefficient (PCC), as explained by Benesty et al. (2009)[13] and computed using equation (19) below, measures linear correlation between two variables. The value obtained for the PCC is in the range between -1 and 1, where: +1 is a total positive linear correlation, 0 indicates that there is no linear association between the two variables, and -1 means total negative linear correlation. In this case, the PCC is approximately 0.422, which is considered a moderate value of correlation.

$$\rho_{X,Y} = \frac{cov_{X,Y}}{\sigma_X \sigma_Y}. \tag{19}$$

Granger-causality tests, proposed in 1969 and explained by Brooks (2008)[7], are done to understand questions such as "Do changes in one variable cause changes in the other variable?". If the answer is positive, then the lags (past values) of the first variable should be significant in the equation for the second variable and it can be stated than the first variable "Granger-causes" the second variable. The meaning of this statement is that there is a correlation between the current value of one variable and the past values of the other, indicating if one time series is useful in forecasting the other. This procedure was replicated and, as the null hypothesis of the test was that one variable does not "Granger-cause" the other, the p-value for the Lower Bound based on the SVIX index "Granger-causing" Equity Risk Premium (ERP) is 0 and for ERP "Granger-causing" the Lower Bound is 0.8482. This means that, in the first case, the null hypothesis is rejected and the Lower Bound "Granger-causes" ERP, for a significance level of 5%. However, ERP does not "Granger-cause" the Lower Bound based on the SVIX index, for the same significance level. In conclusion, the Lower Bound given by the SVIX index is useful for the short-term analysis of the Equity Risk Premium and the SVIX index is indeed an important tool for this.

4. Conclusions

One of the greatest achievements of this dissertation was to check that it is possible to replicate the procedures applied both by CBOE (2018) [10] and Martin (2017) [16] and compute the VIX and the SVIX indexes. The SVIX index measures the risk-neutral volatility of the return on the market while VIX index considers out-of-the-money puts with a higher weight than out-of-the-money calls, highlighting left-tail events.

It is also shown that the approach proposed by Martin (2017)[16], after being put into practice, can yield similar results and conclusions. Besides the fact that not all the same inputs were used (for example, a Nelson Siegel yield curve was used here) and that the computations were not done for the exact same time period, the trends are similar. The VIX index was computed from January 2015 to April 2019 and the results were very close to the ones offered by CBOE. The Lower Bound for a 12-month horizon was compared, on a monthly basis, from January 2015 to April 2019, with the Equity Risk Premium using graphs, the Pearson correlation coefficient and Granger-causality tests. Based on the results of the mentioned econometric indicators, it is possible to confirm that the Lower Bound Granger-causes the Equity Risk Premium, which means that the past values of the SVIX index can be useful to forecast the Equity Risk Premium. This conclusion is aligned with the thesis that the SVIX index is a predictor of the market risk premium.

A. Appendix

A.1. The Negative Correlation Condition (NCC)

The Negative Correlation Condition holds if $cov_t(M_T R_T, R_T) \leq 0$. This is extremely important for the results obtained by Martin (2017)[16]. It is also worth of mention that the NCC holds under some conditions but these conditions are not necessary for it to hold (it can hold even without verifying any of those conditions). The conditions are related to some macro-finance models: Campbell and Cochrane (1999)[8], Bansal and Yaron (2004)[4], Bansal et al. (2014)[19], Campbell et al. (2018)[14], Barro (2006)[5] and Wachter (2013)[20]. If M_T were deterministic, in a risk-neutral economy, the NCC would fail. The conditions for it to hold are: the SDF needs to be volatile (Hansen and Jagannathan (1991)[12]) and negatively correlated with the return R_T.

A.2. The VIX index

In subsection 2.1, equation (1) introduces the VIX index calculation. Here, the formula for the continuous-time limit of equation (1) is shown and demonstrated. According to CBOE (2018)[10], the VIX index includes options with prices that reflect the market's expectation of future volatility.

Combining equation (1) with the definition of ΔK_i given by equation (1b),

$$\begin{aligned}\sigma^2(t,T) &= \frac{2}{T-t}\sum_{\substack{i=-n\\ i\neq 0}}^{N} \frac{\Delta K_i}{{K_i}^2}\; e^{r(T-t)}\; O_t(S_t,K_i,T) \\ &+ \frac{2}{T-t}\frac{\Delta K_0}{{K_0}^2}\; e^{r(T-t)}\; \frac{c_t(S_t,K_0,T)\;+\;p_t(S_t,K_0,T)}{2} \\ &- \frac{1}{T-t}\frac{\Big[F(t,T)-K_0\Big]}{{K_0}^2}\;\times\Big[F(t,T)-K_0\Big]\end{aligned} \tag{20}$$

where $O_t(S_t, K_i, T)$ is given by equation (2a). Assuming that,

$$\Delta K_0 \approx F(t, T) - K_0, \tag{21}$$

and using the put-call parity,

$$\begin{aligned} &c_t(S_t, K_0, T) - p_t(S_t, K_0, T) = S_T \; e^{-q \; (T-t)} - K_0 \; e^{-r \; (T-t)} \\ &\Leftrightarrow \; e^{r \; (T-t)} \Big[c_t(S_t, K_0, T) - p_t(S_t, K_0, T) \Big] = F(t, T) - K_0, \end{aligned} \tag{22}$$

then,

$$\begin{aligned} \sigma^2(t, T) &= \frac{2}{T-t} \sum_{\substack{i=-n \\ i \neq 0}}^{N} \frac{\Delta K_i}{{K_i}^2} \; e^{r(T-t)} \; O_t(S_t, K_i, T) \\ &+ \frac{1}{T-t} \frac{\Delta K_0}{{K_0}^2} \; e^{r(T-t)} \Big[c_t(S_t, K_0, T) + p_t(S_t, K_0, T) - c_t(S_t, K_0, T) + p_t(S_t, K_0, T) \Big] \\ &= \frac{2}{T-t} \sum_{i=-n}^{N} \frac{\Delta K_i}{{K_i}^2} e^{r(T-t)} O_t(S_t, K_i, T) \end{aligned} \tag{23}$$

Hence, up to a discriterization error,

$$\sigma^2(t, T) \approx \frac{2}{T-t} e^{r(T-t)} \int_0^\infty \frac{1}{K^2} O_t(S_t, K, T) \; dK. \tag{24}$$

A.3. The SVIX index

Using a notation that is similar to the one used in subsection A.2 of the Appendix, it is possible to arrive at the SVIX index. Considering,

$$\alpha = (e^{r(T-t)})^2 \times {S_t}^2, \tag{25}$$

then, the discrete-time version of the SVIX index is:

$$SVIX^2(t,T) = \frac{2}{(T-t)\alpha} \sum_{i=-n}^{N} \frac{\Delta K_i}{P(t,T)} \, \overline{O}_t(S_t, K_i, T) - \frac{1}{(T-t)\alpha}\Big(F(t,T) - K_0\Big)^2. \tag{26}$$

Using the definitions of $\overline{O_t}(S_t, K_i, T)$ and $O_t(S_t, K_i, T)$, given in equations (1a) and (2a),

$$\begin{aligned} SVIX^2(t,T) = {} & \frac{2}{(T-t)\alpha} \sum_{\substack{i=-n \\ i\neq 0}}^{N} \Delta K_i \; e^{r(T-t)} \; O_t(S_t, K_i, T) \\ & + \frac{2}{(T-t)\alpha} \Delta K_0 \; e^{r(T-t)} \; \frac{c_t(S_t, K_0, T) \; + \; p_t(S_t, K_0, T)}{2} \\ & - \frac{1}{(T-t)\alpha}\Big(F(t,T) - K_0\Big) \; \times \Big(F(t,T) - K_0\Big). \end{aligned} \tag{27}$$

Furthermore, put-call parity and the approximation $\Delta K_0 \approx F(t,T) - K_0$ imply that,

$$\begin{aligned} SVIX^2(t,T) = {} & \frac{2}{(T-t)\alpha} \sum_{\substack{i=-n \\ i\neq 0}}^{N} \Delta K_i \; e^{r(T-t)} \; O_t(S_t, K_i, T) \\ & + \frac{1}{(T-t)\alpha} \Delta K_0 \; e^{r(T-t)} \Big[c_t(S_t, K_0, T) + p_t(S_t, K_0, T) \\ & - c_t(S_t, K_0, T) + p_t(S_t, K_0, T) \Big], \end{aligned} \tag{28}$$

i.e.,

$$SVIX^2(t,T) = \frac{2}{(T-t)\alpha} \sum_{i=-n}^{N} \Delta K_i O_t(S_t, K_i, T) \tag{29}$$

Hence, and up to a discriterization error,

$$SVIX^2(t,T) \approx \frac{2}{(T-t)\alpha} \int_0^{\infty} O_t(S_t, K, T) \; dK \tag{30}$$

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